INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS
Int. J. Numer. Meth. Fluids 2004; **44**:257–275 (DOI: 10.1002/fld.636)

Simulation of transient flow in pipelines for computer-based operations monitoring

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SUMMARY

Computer models can provide the basis for real-time monitoring and control of fluid flow in pipelines. Problems of fluid flow in pipelines are mathematically represented by a non-linear system of coupled partial differential equations. In this paper, several numerical techniques are evaluated with respect to their suitability for the purpose of real-time monitoring of fluid flow in pipelines. The proposed techniques are evaluated in terms of the L_1, the L_2, and the L_∞ error norms. Moreover, the developed simulators will be compared in terms of their speed of response and settling time which are essential factors for an effective real-time monitoring scheme. Finally, the selected simulation scheme is further tested under assumed pipeline leak conditions. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: fluid flow in pipelines; numerical methods; real-time monitoring; pipeline leak simulation

1. INTRODUCTION

The operation of pipelines carrying vital fluids is of prime concern from safety and economic points of view. The safe and optimal operation of these pipelines can be effectively and economically achieved via computer-based operation monitoring with much reduced need for otherwise expensive sensors and hardware. Nowadays, system-based techniques implemented on fast computers can replace the heavily instrumented pipeline configurations, which are costly to install and maintain, in achieving safe and optimal pipeline operation. The success of such techniques for pipelines monitoring and control hinges on the development of reliable

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Contract/grant sponsor: King Fahd University of Petroleum & Minerals (KFUPM)
Contract/grant sponsor: King Abdulaziz City for Science and Technology (KACST); contract/grant number: AR-17-34

Received 24 June 2002
Revised 24 February 2003

computer simulation models. This paper is aimed at developing computer models suitable for online monitoring of pipeline operations.

It has become evident that real-time monitoring and leak detection schemes in pipelines, based on transient flow models, are gaining interest among researchers in recent years. Several authors have addressed the numerical solution of the problem of fluid flow in pipelines for different purposes. For example, transients in natural gas pipelines were investigated using the classical method of characteristics by Stoner [1], and Yow [2]. Thompson and Skogman [3] addressed the problem of real-time flow modelling and emphasized its potential for effective monitoring schemes. However, they did not evaluate any specific numerical scheme in that regard. Liou and Tian [4] considered pipeline monitoring for leak detection on the basis of the classical method of characteristics. Also, Choy *et al.* [5] used numerical techniques to study pressure transients in fluid pipelines resulting from valve closures. Based on their findings, they concluded that models based on finite difference techniques could be more efficient than those based on the classical method of characteristics, and at the same time are numerically stable. The success of real time monitoring schemes based on transient flow models is highly dependent on performance characteristics like computational accuracy, speed of response, and settling time. With that in mind, this paper aims at presenting comparative investigations to evaluate the computational performance of several numerical techniques as applied to transient flow in pipelines. The paper also provides measures by which one can select a computer model suitable for effective online monitoring of pipelines operating under fast dynamic conditions and the possibility of occurrence of a sudden event of a small magnitude such as a leak.

2. DEVELOPMENT OF PIPELINE COMPUTER SIMULATION MODELS

2.1. Physical modeling of fluid flow in a pipeline

We consider the flow of a compressible fluid in a straight segment of a pipeline, with constant cross-sectional area 'A', where the pipe wall friction being the only contributor to pressure drop through the pipe. The friction factor 'f' is assumed to be constant along the length of the pipe, and the pressure drop due to friction is given by Darcy–Weisbach formula. Further, we assume a one-dimensional isothermal flow with velocity much less than the acoustic velocity. Applying the mass and momentum balance conditions on a control volume of length dz and with the same cross section as the pipe, one gets the following PDE representing the dynamics of gas flow in the pipeline,

$$p_t + \frac{a^2}{A} q_z = 0 \tag{1}$$

$$q_t + Ap_z + \frac{fa^2}{2AD}\frac{q|q|}{p} + \frac{Ag\sin\psi}{a^2} p = 0 \tag{2}$$

where $p \equiv p(t,z)$ is the pressure at location z and time t, $q \equiv q(t,z)$ is the flow rate at location z and time t, and the subscript t stands for the first derivative with respect to time, and the subscript z stands for the first derivative with respect to distance along the pipeline axis. Details of the derivation of this model are as given in Appendix A. Equations (1) and (2) are given for no leak conditions (i.e. $q_{\mathrm{L}} = 0$).

The wave speed a in the above equations accounts for the pipeline elastic effects as well as the fluid physical properties as given by the following expression [6].

$$a = \sqrt{\frac{K/\rho}{1 + KD(1-\mu^2)/E\tau}} \tag{3}$$

The above wave speed expression is obtained from the basic derivation of the mass conservation equation of unsteady flow when the pipe elastic deformations are considered.

The initial and boundary conditions of the above problem are stated as follows:

$$\begin{aligned} p(z,0) &= p(z) \quad \text{initial pressure distribution} \\ q(z,0) &= q(z) \quad \text{initial flow distribution} \\ p(0,t) &= p_0 \quad \text{inlet pressure} \\ p(L,t) &= p_e \quad \text{exit pressure} \end{aligned}$$

In order to apply the above physical model to the problem of computer-based online monitoring of gas flow, one needs to employ a suitable numerical scheme. Such a numerical scheme must be able to simulate the system response due to fast dynamics, which can result from normal operational conditions, as well as being accurate enough to be able to detect events of relatively small magnitudes. Therefore, in this paper we are going to evaluate several available numerical techniques in that regard. Measures will be developed and used to evaluate such techniques as related to their accuracy as well as their ability to track system transient following fast dynamic operating condition.

2.2. Discretization techniques

Consider a pipe of length L, cross-sectional area A, inner diameter D, and constant slope ψ. For simplicity of notation, let

$$r_1 = \frac{a^2\Delta t}{A\Delta z}, \quad r_2 = \frac{A\Delta t}{\Delta z}, \quad r_3 = \frac{fa^2\Delta t}{2AD}, \quad r_4 = \frac{Ag\sin\psi\Delta t}{a^2} \tag{4}$$

2.2.1. Alternating-space methods. In this scheme, space derivatives are evaluated using an alternating discretization procedure. The space derivative of pressure is evaluated using forward difference and that of flow rate is evaluated using backward difference. The time-derivatives of both pressure and flow rate, however, are evaluated using backward difference. This discretization is motivated by physical insight where pressure differential is the driving force for the flow, and by the fact that in many practical situations, such as monitoring of pipeline leaks, the exit pressure rather than the exit flow is known. Therefore, the involved partial derivatives are substituted by

$$\left(\frac{\partial p}{\partial t}\right)_i^{t+\theta} = \frac{p_i^{t+1} - p_i^t}{\Delta t} + O(\eta)$$

$$\left(\frac{\partial q}{\partial t}\right)_i^{t+\theta} = \frac{q_i^{t+1} - q_i^t}{\Delta t} + O(\eta)$$

$$
\begin{aligned}
\left(\frac{\partial p}{\partial z}\right)_i^{t+\theta} &= \theta\,\frac{p_{i+1}^{t+1}-p_i^{t+1}}{\Delta z} + (1-\theta)\,\frac{p_{i+1}^{t}-p_i^{t}}{\Delta z} + O(\Delta z) \\
\left(\frac{\partial q}{\partial z}\right)_i^{t+\theta} &= \theta\frac{q_i^{t+1}-q_{i-1}^{t+1}}{\Delta z} + (1-\theta)\,\frac{q_i^{t}-q_{i-1}^{t}}{\Delta z} + O(\Delta z)
\end{aligned} \tag{5}
$$

After some manipulations, the differential equations (1) and (2) are transformed into the following difference equations:

$$p_i^{t+1} + r_1\theta q_i^{t+1} - r_1\theta q_{i-1}^{t+1} = p_i^t - r_1(1-\theta)q_i^t + r_1(1-\theta)q_{i-1}^t \tag{6}$$

$$
\begin{aligned}
q_i^{t+1} + r_2\theta p_{i+1}^{t+1} - r_1\theta p_i^{t+1} = q_i^t &- r_1(1-\theta)p_{i+1}^t + r_1(1-\theta)p_i^t \\
&- r_3\,\frac{q_i^t|q_i^t|}{p_i^t} - r_4\,p_i^t
\end{aligned} \tag{7}
$$

It is noteworthy to point out that when $\theta=0$ the above discretization scheme is an explicit forward-time alternating-space (FTAS) scheme of first order, i.e. $\eta=\Delta t$. And for $\theta=1$ it is an implicit backward-time alternating-space (BTAS) first order, i.e. $\eta=\Delta t$, scheme. While for $\theta=0.5$, the discretization scheme is an implicit centred-time alternating-space (CTAS) scheme of second order, i.e. $\eta=(\Delta t/2)^2$. Substituting Equations (5) into Equations (1), and (2) it follows that Equations (6) and (7) above have numerical errors of order $O((\Delta t)^3/4, \Delta z\Delta t)$ for $\theta=0.5$.

2.2.2. Centred-space methods. In this finite-difference scheme, the space derivatives of both pressure and flow are evaluated using central difference. The time derivatives of both pressure and flow rate are evaluated using backward difference. Therefore, the partial derivatives of Equations (1) and (2) are approximated as follows:

$$
\begin{aligned}
\left(\frac{\partial p}{\partial t}\right)_i^{t+\theta} &= \frac{p_i^{t+1}-p_i^t}{\Delta t} + O(\eta) \\
\left(\frac{\partial q}{\partial t}\right)_i^{t+\theta} &= \frac{q_i^{t+1}-q_i^t}{\Delta t} + O(\eta) \\
\left(\frac{\partial p}{\partial z}\right)_i^{t+\theta} &= \theta\,\frac{p_{i+1}^{t+1}-p_{i-1}^{t+1}}{2\Delta z} + (1-\theta)\,\frac{p_{i+1}^{t}-p_{i-1}^{t}}{2\Delta z} + O(\Delta z^2) \\
\left(\frac{\partial q}{\partial z}\right)_i^{t+\theta} &= \theta\,\frac{q_{i+1}^{t+1}-q_{i-1}^{t+1}}{2\Delta z} + (1-\theta)\,\frac{q_{i+1}^{t}-q_{i-1}^{t}}{2\Delta z} + O(\Delta z^2)
\end{aligned} \tag{8}
$$

Therefore, the governing partial differential equations can be approximated by the following difference equations:

$$p_i^{t+1} + \frac{r_1}{2}\,\theta q_{i+1}^{t+1} - \frac{r_1}{2}\,\theta q_{i-1}^{t+1} = p_i^t - \frac{r_1}{2}\,(1-\theta)(q_{i+1}^t - q_{i-1}^t) \tag{9}$$

$$q_i^{t+1} + \frac{r_2}{2}\,\theta p_{i+1}^{t+1} - \frac{r_2}{2}\,\theta p_{i-1}^{t+1} = q_i^t - \frac{r_2}{2}\,(1-\theta)\,p_{i+1}^t + \frac{r_2}{2}\,(1-\theta)\,p_{i-1}^t$$
$$-r_3\,\frac{q_i^t|q_i^t|}{p_i^t} + r_4\,p_i^t \qquad (10)$$

The constant parameter θ is used to control the numerical scheme to give the following alternatives;

- For $\theta = 0$, an explicit forward-time centred-space (FTCS) scheme of first order $\eta = \Delta t$;
- For $\theta = 1.0$, an implicit backward-time centred-space (BTCS) scheme, a Crank–Nicolson scheme [7], of first order $\eta = \Delta t$;
- And, for $\theta = 0.5$, an implicit centred-time centred-space (CTCS) scheme, alternative Crank–Nicolson scheme [7], of second order $\eta = (\Delta t/2)^2$.

Substituting Equations (8) into Equations (1), and (2) it follows that Equations (9) and (10) above have numerical errors of order $O((\Delta t)^3/4, (\Delta z)^2 \Delta t)$ for $\theta = 0.5$.

2.2.3. A predictor–corrector method. A predictor–corrector technique can be based on the method of MacCormack (MC), which involves the propagation of the numerical solution in time using Taylor's series in conjunction with space-derivative approximations using both forward and backward finite differences [8]. The method is employed here to develop a predictor–corrector simulation scheme for the pipeline flow problem at hand. Consider the pipeline to be segmented by equally spaced nodes separated by segments of equal length Δz. Also, consider the time to be discretized by time increments of Δt. Let p_i^t, q_i^t be the computed pressure, and flow rate at node i and time step t. Employing a second order Taylor's series expansion, the solution can be advanced to time step $t+1$ as follows:

$$p_i^{t+1} = p_i^t + \left(\frac{\partial p}{\partial t}\right)_i^t \Delta t + \frac{1}{2}\left(\frac{\partial^2 p}{\partial t^2}\right)_i^t \Delta t^2 + O(\Delta t^3) \qquad (11)$$

where the second order partial derivative is approximated by

$$\left(\frac{\partial^2 p}{\partial t^2}\right)_i^t = \frac{(\partial p/\partial t)_i^{t+1} - (\partial p/\partial t)_i^t}{\Delta t} + O(\Delta t) \qquad (12)$$

Substituting for the first partial derivative of the pressure with respect to time using Equation (1), the solution of Equation (11) can then be expressed as

$$p_i^{t+1} = p_i^t - \frac{1}{2}\,\frac{a^2}{A}\left[\left(\frac{\partial q}{\partial z}\right)_i^{t+1} + \left(\frac{\partial q}{\partial z}\right)_i^t\right]\Delta t + O(\Delta t^3) \qquad (13)$$

Further, the first partial derivatives of the flow rate with respect to space are approximated by

$$\left(\frac{\partial q}{\partial z}\right)_i^{t+1} = \frac{\bar{q}_i^{t+1} - \bar{q}_{i-1}^{t+1}}{\Delta z} + O(\Delta z) \quad \text{and} \quad \left(\frac{\partial q}{\partial z}\right)_i^t = \frac{q_{i+1}^t - q_i^t}{\Delta z} + O(\Delta z) \qquad (14)$$

Therefore, the computed pressure is advanced in time according to,

$$p_i^{t+1} = p_i^t - \frac{1}{2} r_1[(q_{i+1}^t - q_i^t) + (\bar{q}_i^{t+1} - \bar{q}_{i-1}^{t+1})] + O((\Delta t)^3, \Delta z \Delta t) \tag{15}$$

where, the values with over bar are predicted according to,

$$\bar{q}_i^{t+1} = q_i^t - r_2(p_{i+1}^t - p_i^t) - r_3 \left(\frac{q_i^t |q_i^t|}{p_i^t} \right) - r_4 p_i^t + O((\Delta t)^2, \Delta z \Delta t) \tag{16}$$

Similarly, advancing the solution for the flow rate,

$$q_i^{t+1} = q_i^t + \left(\frac{\partial q}{\partial t} \right)_i^t \Delta t + \frac{1}{2} \left(\frac{\partial^2 q}{\partial t^2} \right)_i^t \Delta t^2 + O(\Delta t^3) \tag{17}$$

where

$$\left(\frac{\partial^2 q}{\partial t^2} \right)_i^t = \frac{(\partial q/\partial t)_i^{t+1} - (\partial q/\partial t)_i^t}{\Delta t} + O(\Delta t) \tag{18}$$

Substitute for the first partial derivative of flow rate with respect to time using Equation (2), then substitute for the pressure space derivatives using

$$\left(\frac{\partial p}{\partial z} \right)_i^{t+1} = \frac{\bar{p}_i^{t+1} - \bar{p}_{i-1}^{t+1}}{\Delta z} + O(\Delta z) \quad \text{and} \quad \left(\frac{\partial p}{\partial z} \right)_i^t = \frac{p_{i+1}^t - p_i^t}{\Delta z} + O(\Delta z) \tag{19}$$

Finally, the computed flow rate is advanced in time according to

$$\begin{aligned} q_i^{t+1} = q_i^t &- \frac{1}{2} r_2[(p_{i+1}^t - p_i^t) + (\bar{p}_i^{t+1} - \bar{p}_{i-1}^{t+1})] \\ &- \frac{1}{2} r_3 \left(\frac{q_i^t |q_i^t|}{p_i^t} + \frac{\bar{q}_i^{t+1} |\bar{q}_i^{t+1}|}{\bar{p}_i^{t+1}} \right) - \frac{1}{2} r_4(p_i^t + \bar{p}_i^{t+1}) + O((\Delta t)^3, \Delta z \Delta t) \end{aligned} \tag{20}$$

And the predicted values of the pressure are given by

$$\bar{p}_i^{t+1} = p_i^t - r_1(q_{i+1}^t - q_i^t) + O((\Delta t)^2, \Delta z \Delta t) \tag{21}$$

In summary, the predictor equations are given by

$$\bar{p}_i^{t+1} = p_i^t - r_1(q_{i+1}^t - q_i^t) \tag{22}$$

$$\bar{q}_i^{t+1} = q_i^t - r_2(p_{i+1}^t - p_i^t) - r_3 \left(\frac{q_i^t |q_i^t|}{p_i^t} \right) - r_4 p_i^t \tag{23}$$

where the predictor's numerical accuracy is $O((\Delta t)^2, \Delta z \Delta t)$. The corrector equations are given by

$$p_i^{t+1} = p_i^t - \tfrac{1}{2} r_1[(q_{i+1}^t - q_i^t) + (\bar{q}_i^{t+1} - \bar{q}_{i-1}^{t+1})] \tag{24}$$

$$q_i^{t+1} = q_i^t - \frac{1}{2} r_2[(p_{i+1}^t - p_i^t) + (\bar{p}_i^{t+1} - \bar{p}_{i-1}^{t+1})]$$
$$- \frac{1}{2} r_3 \left(\frac{q_i^t |q_i^t|}{p_i^t} + \frac{\bar{q}_i^{t+1} |\bar{q}_i^{t+1}|}{\bar{p}_i^{t+1}} \right) - \frac{1}{2} r_4 (p_i^t + \bar{p}_i^{t+1}) \quad (25)$$

where the corrector's numerical accuracy is $O(\Delta t^3, \Delta t \Delta z)$.

3. NUMERICAL SIMULATIONS

In this paper, three numerical schemes are used to develop computer models to simulate the transient flow of gas in a pipeline. The physical model of such a problem is non-linear as given by Equations (1) and (2). Therefore, one could not simply rely on the known orders of accuracy of the discretization schemes. It becomes necessary to test the accuracy of the models via numerical simulations. In conducting such numerical simulations, a horizontal straight pipeline is considered with data as listed in Table I.

3.1. Flow rate and pressure computation

The computed pressure profiles along the pipeline, as obtained by using the three simulation models developed in this paper, are shown in Figure 1. Similarly, the computed flow rate profiles are given in Figure 2. Both pressure and flow rate profiles are presented at several times, in order to show the growth of these profiles towards their respective steady state profiles. Also, the computed time-response of both pressure and flow rate are as displayed in Figures 3 and 4, respectively, at several nodal points. These results indicate that the three developed simulation models are capable of providing numerically stable solutions for the pipeline fluid flow problem. It is noticed that the CTCS method exhibits some oscillations in the steady state flow rate solution. The magnitude of such oscillations diminishes as the discretization grid is refined.

3.2. Characterization of the simulators numerical performance

To facilitate the quantitative comparison of the developed numerical simulators, the L_1-norm, L_2-norm, and the L_∞-norm of their respective computational errors, as defined next, will be evaluated.

Table I. Pipeline parameters.

Parameter	Value
Pipeline length	90 km
Pipe cross-sectional area	1 m^2
Wave speed	300 m/s
Pipe wall friction coefficient	0.003
Inlet pressure	1.5 bar
Outlet pressure	1.0 bar
Inlet pressure step, P_s	2.0 bar

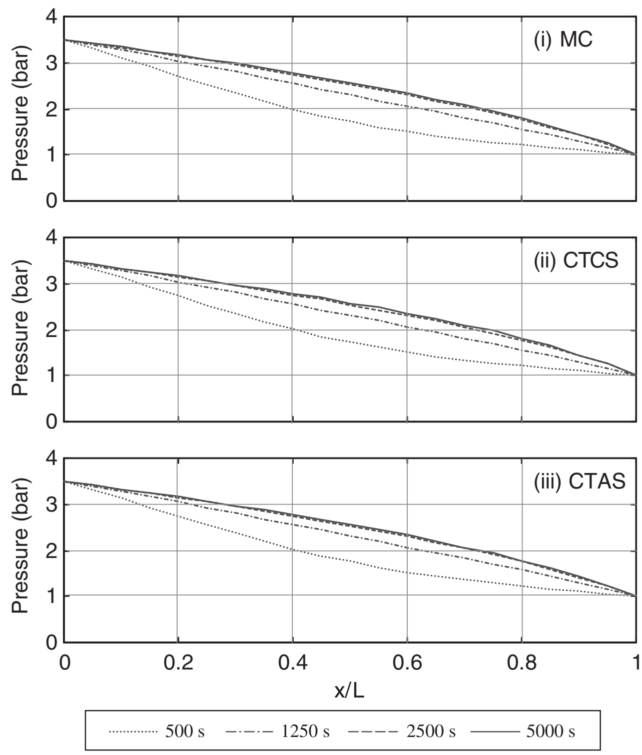


Figure 1. Pressure profiles using different methods.

$$
\begin{aligned}
E_1^t &= \frac{1}{N}\sum_{i=1}^{N}\frac{|X_i^{\mathrm{ss}} - X_i^t|}{X_o}\\
E_2^t &= \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{X_i^{\mathrm{ss}} - X_i^t}{X_o}\right)^2}\\
E_3^t &= \operatorname*{Max}_{1\leqslant i\leqslant N}\left(\frac{|X_i^{\mathrm{ss}} - X_i^t|}{X_o}\right)
\end{aligned}
\tag{26}
$$

The symbols used in Equation (26) are defined as

$X_i^t \equiv$ The computed value of pressure or flow rate at the ith node and time t.
$X_i^{\mathrm{ss}} \equiv$ The true steady-state value of pressure or flow rate at the ith node.
$X_o \equiv$ A normalizing factor, which is defined as follows:
$X_o = q^{\mathrm{ss}}$; for the flow rate calculations, and
$X_o = P_{\mathrm{s}}$; for pressure calculations,

where, q^{ss} represents the steady-state value of the flow rate through the pipeline, and P_{s} is the value of the applied input pressure step. In the steady state, Equations (1) and (2)

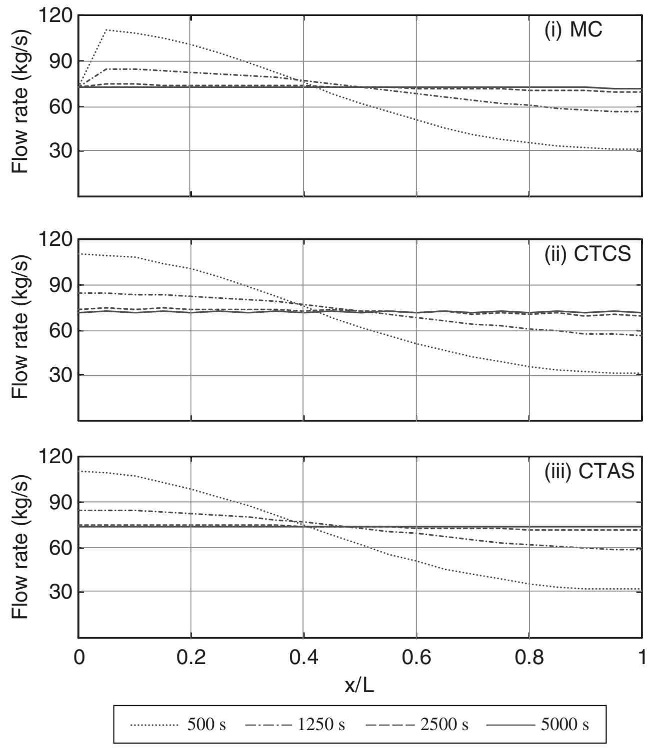


Figure 2. Flow rate profiles using different methods.

reduce to

$$\frac{\mathrm{d}q}{\mathrm{d}z} = 0 \tag{27}$$

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\frac{fa^2}{2A^2D}\frac{q^2}{p} \tag{28}$$

Hence, in the steady state the flow rate is constant along the pipeline, and the pressure profile is such that p^2 varies linearly with z. And, upon integration of Equation (28) using the following boundary conditions:

$$\begin{aligned} p &= P_o \quad \text{at } z = 0, \quad \text{and} \\ p &= P_N \quad \text{at } z = L \end{aligned} \tag{29}$$

The steady-state flow rate q^{ss} is obtained as

$$q^{\mathrm{ss}} = \sqrt{\frac{A^2D}{fa^2}\left(\frac{P_0^2 - P_N^2}{L}\right)} \tag{30}$$

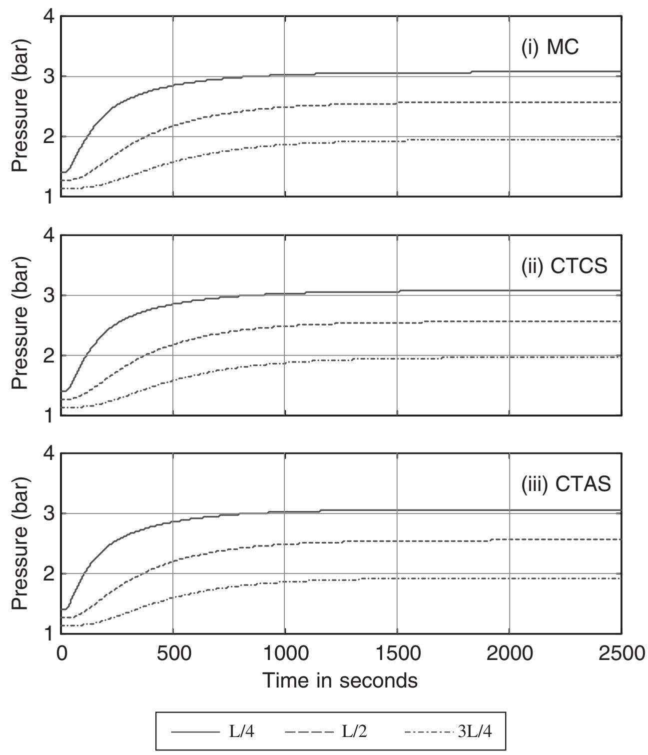


Figure 3. Pressure time-response at locations $L/4$, $L/2$ and $3L/4$.

Values of the error norms for the three methods are given in Tables II, and III, for three different grid sizes. The time histories of the errors are as shown in Figures 5, and 6. The results indicate that, the error norms for the three methods are more sensitive to space discretization than time discretization. Moreover, the results indicate that both CTCS, and MC methods are superior in their numerical accuracy to the CTAS method. It is also noticed that refining the mesh size, doubling the number of nodes, results in a pronounced improvement in the accuracy of the CTCS method, which approaches the level of accuracy of the MC method.

Other measures of the transient performance characteristics for the three numerical schemes are given in terms of the *rise time* and *settling time*. The rise time is defined as the time taken by each method for the computed flow rate at the end node to reach 90% of its steady-state value. The settling time, however, is characterized as the time taken by each method for the computational errors, in terms of the error norm E_3, the most conservative of the three error norms, to settle within 5% envelop. These time-response characteristics are displayed in Table IV for the different methods with three different mesh sizes in each case.

It is interesting to note that the CTAS attains a relatively shorter rise time and settling time than the other two methods. The faster response characteristics of the CTAS, however, are penalized by lower accuracy. Both the CTCS and MC methods attain nearly the same rise time and settling time, as displayed in Table IV.

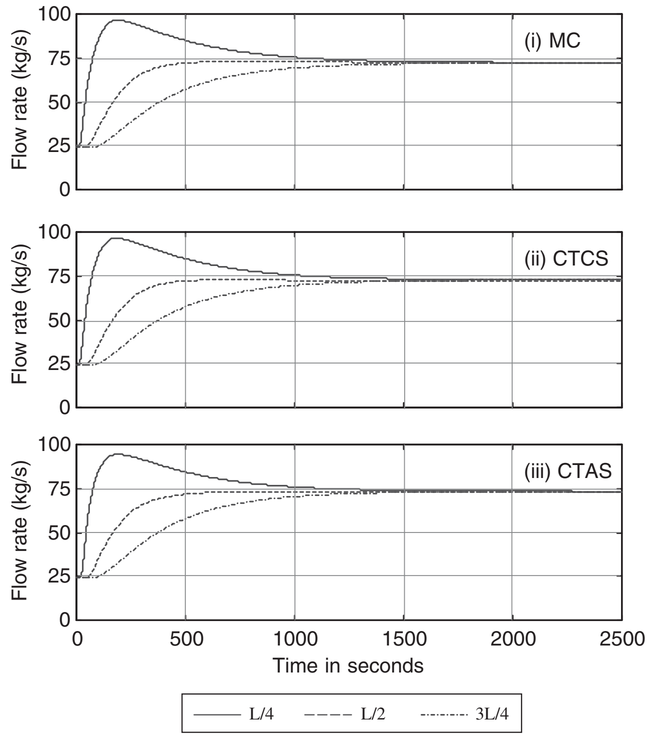


Figure 4. Flow rate time-response at locations $L/4$, $L/2$ and $3L/4$.

Table II. Pressure error norms.

Error norm	E_1	E_2	E_3
Case (1): $N=21$, $\Delta t=2$ s			
CTAS	0.656	0.168	1.190
CTCS	0.491	0.132	0.660
MC	0.240	0.061	0.410
Case (2): $N=41$, $\Delta t=2$ s			
CTAS	0.460	0.0810	0.7680
CTCS	0.140	0.0302	0.3565
MC	0.153	0.0270	0.2410
Case (3): $N=41$, $\Delta t=1$ s			
CTAS	0.456	0.0812	0.7680
CTCS	0.1405	0.0301	0.3554
MC	0.1560	0.0270	0.2440

Table III. Flow rate error norms.

Scheme	E_1	E_2	E_3
Case (1): $N = 21$, $\Delta t = 2$ s			
CTAS	1.526	0.333	1.526
CTCS	0.682	0.148	0.684
MC	0.234	0.065	0.82
Case (2): $N = 41$, $\Delta t = 2$ s			
CTAS	0.77	0.1225	0.9786
CTCS	0.21	0.041	0.5048
MC	0.203	0.0397	0.804
Case (3): $N = 41$, $\Delta t = 1$ s			
CTAS	0.765	0.1223	0.978
CTCS	0.2099	0.0408	0.5032
MC	0.180	0.033	0.478

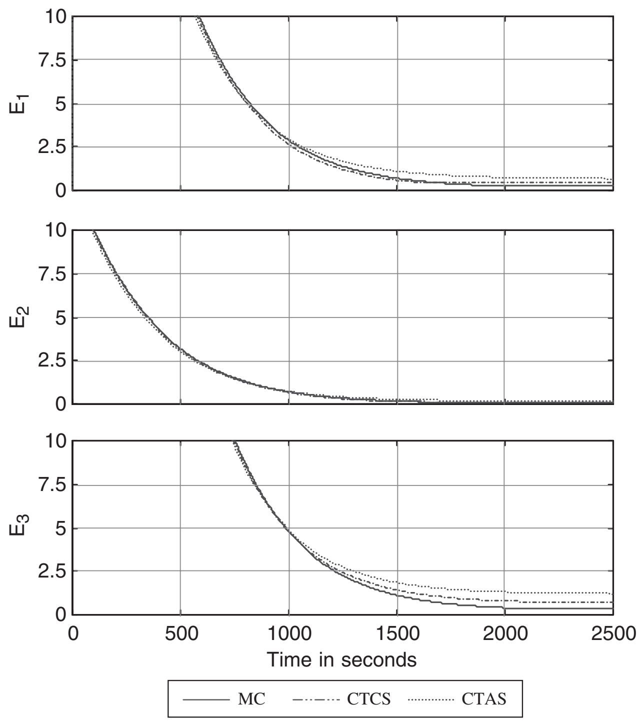


Figure 5. Error norms for computed pressure vs time.

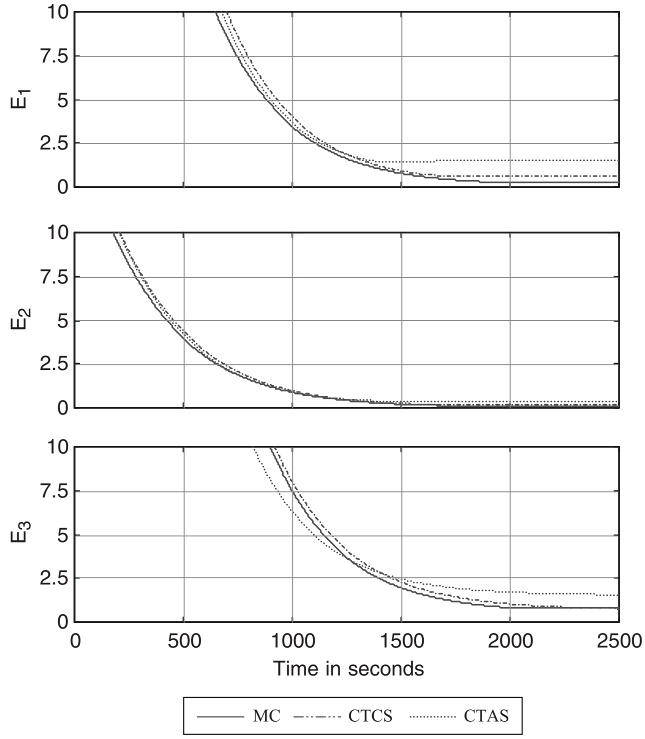


Figure 6. Error norms for computed flow rate vs time.

Table IV. Time response characteristics.

Scheme	Rise time	Settling time	Rise time	Settling time	Rise time	Settling time
	$N=21,\ \Delta t=2$ s		$N=41,\ \Delta t=2$ s		$N=41,\ \Delta t=1$ s	
CTAS	1616	2202	1688	2114	1686	2111
CTCS	1836	2348	1786	2252	1784	2249
MC	1798	2282	1786	2262	1778	2237

4. SIMULATION OF A PIPELINE INCLUDING A LEAK

4.1. Pipeline physical model including a leak

As the main objective is the development of an on-line monitoring capability, the MC based simulator is used to investigate the effects of a leak occurring in the pipeline. Consider an elemental length Δz of the pipe at a distance z from one end with the same cross-section as the pipe. Assume that a leak of magnitude q_L is present in this element. The dynamic model

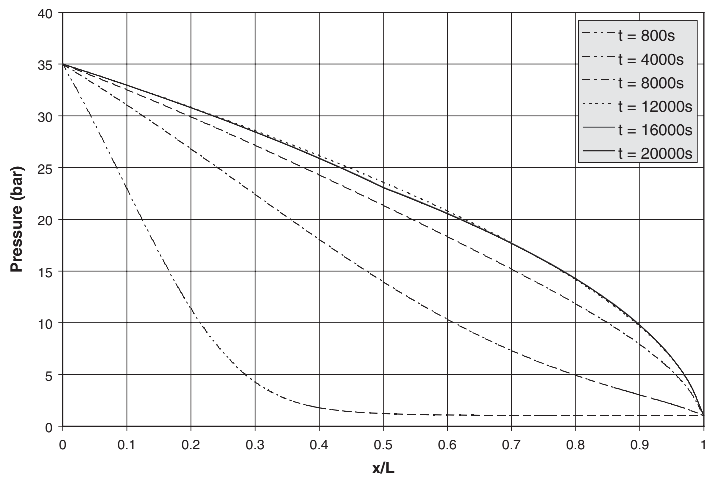


Figure 7. Pressure profile (leak onset at 12 000 s, $A_{\text{leak}}/A = 0.003$).

of the gas flow in this case is given by

$$\frac{\partial p}{\partial t} + \frac{a^2}{A}\frac{\partial q}{\partial z} + \frac{a^2}{A\Delta z} q_{\mathrm{L}} = 0 \tag{31}$$

$$\frac{\partial q}{\partial t} + A\frac{\partial p}{\partial z} - \frac{a^2}{A\Delta z}\left(\frac{q}{p}\right) q_{\mathrm{L}} = -\frac{p}{a^2} Ag \sin\psi - \frac{f}{2D}\frac{a^2}{A}\frac{q|q|}{p} \tag{32}$$

Details of the derivation of this model are given in Appendix A.

4.2. *Investigation of leak effects via simulation*

The pipeline simulation example presented in Section 3 is considered here under the influence of a leak. The effects of a 10% leak occurring at 1200 s while the flow is still in the transient phase are investigated. Figure 7 shows the progression of the pressure profile, while Figure 8 gives a graphical comparison of the steady state pressure profiles under no leak and leak conditions. It is clear that the simulation captures the effect of the leak as the pressure profile is lowered all over the pipeline length. Figure 9 shows the progression of the flow rate profile from the transient phase all the way to the steady state under leakage conditions. Figure 10 shows the time response of the flow rate at two nodes, one upstream of the leak and the other down stream of the leak. Again the simulation vividly captures the effect of the leak on the flow pattern. As the pressure gets lower at the leak location, the upstream flow rate is increased while the down stream flow rate is decreased. The net difference between the flow rates on both sides of the leak equals the magnitude of the leak flow rate.

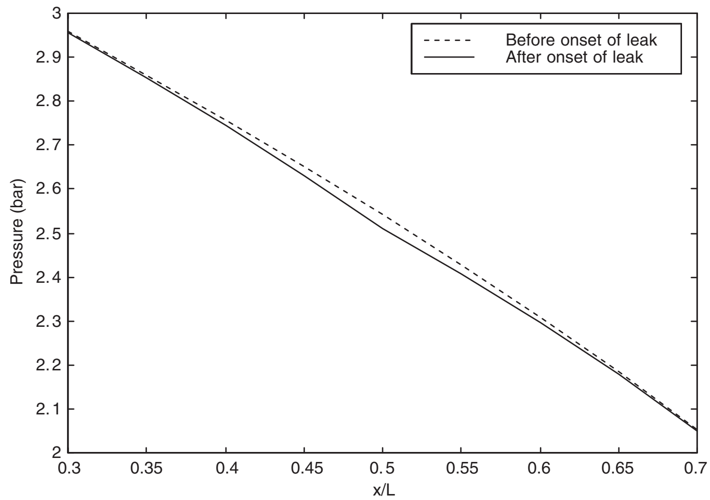


Figure 8. Pressure profile before and after the onset of leak.

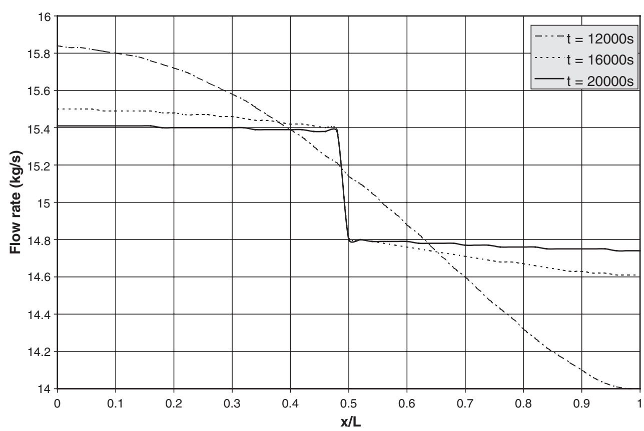


Figure 9. Flow rate profile with leak onset at $t = 12\,000$ s.

5. CONCLUSIONS

Real time pipeline simulators can be developed on the basis of the numerical schemes presented in this paper as these schemes provide stable simulations with suitably fast response. The CTCS method showed small oscillations around the true steady state value of the flow rate. Such oscillations resulted from the accumulated numerical errors at the end node, which gradually propagated backward through the pipeline. The amplitude of such oscillations is reduced as the number of nodes is increased. The CTAS method while being free of oscillating error, it exhibits a residual error, i.e. steady-state bias, in its computed flow rate. The MC method gives the most accurate results of the three methods. The results also show that the accuracy of the CTCS approaches the accuracy of the MC method, as the mesh size is refined.

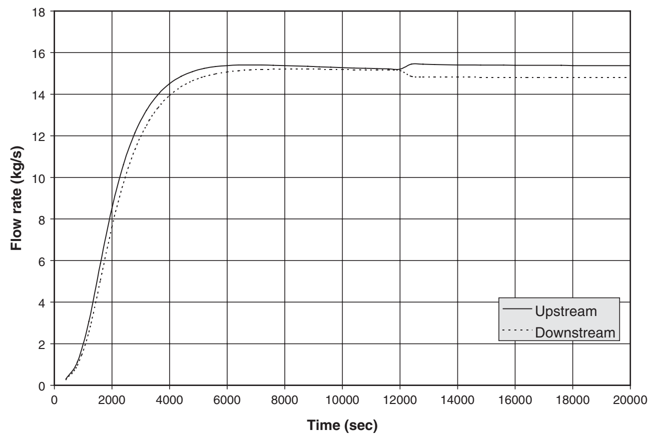
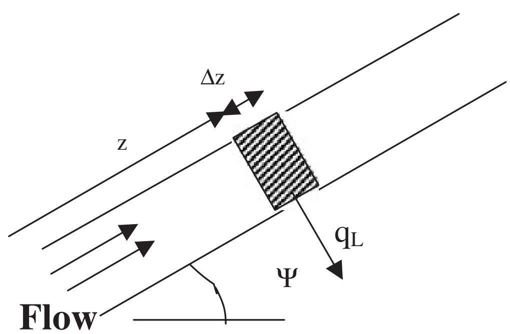


Figure 10. Flow rate response with leak onset at $t = 12\,000$ s.

The MC based simulator is further tested in terms of its suitability as an on-line monitoring scheme for pipeline operation. The simulator is shown to be capable of effectively capturing the effects of a small leak and at the same time having a reasonably fast dynamic response.

APPENDIX A: DERIVATION OF THE PIPELINE FLOW PHYSICAL MODEL



Consider an elemental length Δz of the pipe at a distance z from the inlet. Assume that a leak of magnitude q_L is present in this element. The effect of this leak on the continuity and momentum equations is obtained as follows:

A.1. Continuity equation

By mass balance across the element of length Δz, we have

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \dot{m}_{\text{accumulated}}$$

$$\dot{m}_{\text{in}} = (\rho A w)_z$$

$$\dot{m}_{\text{out}} = (\rho A w)_z + \frac{\partial}{\partial z}(\rho A w)_z \Delta z + q_{\text{L}}$$

$$\dot{m}_{\text{accumulated}} = \frac{\partial}{\partial t}(\rho A \Delta z)$$

$$(\rho A w)_z - \left((\rho A w)_z + \frac{\partial}{\partial z}(\rho A w)_z \Delta z + q_{\text{L}}\right) = \frac{\partial}{\partial t}(\rho A \Delta z)$$

$$A\Delta z \frac{\partial \rho}{\partial t} + A\Delta z \frac{\partial(\rho w)}{\partial z} + q_{\text{L}} = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial z} + \frac{q_{\text{L}}}{A\Delta z} = 0$$

Since $a^2 = p/\rho$, we have $\rho = p/a^2$.
Also, $q = \rho A w \Rightarrow \rho w = q/A$

$$\frac{\partial}{\partial t}\left(\frac{p}{a^2}\right) + \frac{\partial}{\partial z}\left(\frac{q}{A}\right) + \frac{1}{A\Delta z} q_{\text{L}} = 0$$

$$\frac{\partial p}{\partial t} + \frac{a^2}{A}\frac{\partial q}{\partial z} + \frac{a^2}{A\Delta z} q_{\text{L}} = 0$$

A.2. Momentum balance

Rate of change of momentum $= \Sigma$ Forces

$$\dot{M}_{\text{in}} - \dot{M}_{\text{out}} + \dot{M}_{\text{element}} = G + P + F$$

$$\dot{M}_{\text{in}} = (\rho A w).w$$

$$\dot{M}_{\text{out}} = \rho A w.w + \frac{\partial}{\partial z}(\rho A w.w)\,\mathrm{d}z + q_{\text{L}}.w$$

$$\dot{M}_{\text{element}} = \frac{\partial(\rho A \Delta z.w)}{\partial t} = \frac{\partial(\rho w)}{\partial t} A\Delta z$$

$$G = -\rho A \Delta z g \sin\psi$$

$$P = pA - \left(p + \frac{\partial p}{\partial z}\Delta z\right)A = -\frac{\partial p}{\partial z} A\Delta z$$

$$F = -\frac{\rho f w^2}{2D} A\Delta z$$

$$(\rho A w).w - \left(\rho A w.w + \frac{\partial}{\partial z}(\rho A w.w)\,\mathrm{d}z + q_{\text{L}}.w\right) + \frac{\partial(\rho w)}{\partial t} A\Delta z$$

$$= -\rho A \Delta z g \sin\psi - \frac{\partial p}{\partial z} A\Delta z - \frac{\rho f w^2}{2D} A\Delta z$$

$$\frac{\partial(\rho w)}{\partial t}A\Delta z+\frac{\partial p}{\partial z}A\Delta z-\frac{\partial}{\partial z}(\rho w^2).A\Delta z-q_{\mathrm{L}}.w=-\rho A\Delta z g\sin\psi-\frac{\rho f w^2}{2D}A\Delta z$$

$$\frac{\partial(\rho w)}{\partial t}+\frac{\partial p}{\partial z}-\frac{\partial}{\partial z}(\rho w^2)-\frac{q_{\mathrm{L}}.w}{A\Delta z}=-\rho g\sin\psi-\frac{\rho f w^2}{2D}$$

Assuming the flow velocity w to be much less than the acoustic velocity, $(w \ll a)$, the term $\partial(\rho w^2)/\partial z$ becomes negligibly small and can be ignored. Also, expressing w in terms of flow rate q and expressing ρ in terms of p, we get

$$\frac{1}{A}\frac{\partial q}{\partial t}+\frac{\partial p}{\partial z}-\frac{q_{\mathrm{L}}.w}{A\Delta z}=-\frac{p}{a^2}g\sin\psi-\frac{f}{2D}\frac{a^2}{A^2}\frac{q|q|}{p}$$

$$\frac{\partial q}{\partial t}+A\frac{\partial p}{\partial z}-\frac{a^2}{A\Delta z}\left(\frac{q}{p}\right)q_{\mathrm{leak}}=-\frac{p}{a^2}Ag\sin\psi-\frac{f}{2D}\frac{a^2}{A}\frac{q|q|}{p}$$

It should be noted that the derivation of the physical model given by Equations (1) and (2) follows the same steps in this appendix with q_{L} set equal zero.

APPENDIX B

Nomenclature

a	wave speed (m/s)
A	cross-sectional area of the pipe (m^2)
L	length of pipeline (m)
D	pipe diameter (m)
E	elasticity modulus of the pipeline material ($\mathrm{N/m^2}$)
f	friction coefficient
g	acceleration of gravity ($\mathrm{m/s^2}$)
K	bulk modulus ($\mathrm{N/m^2}$)
p	fluid pressure (bar)
q	fluid flow rate (kg/s)
t	time (s)
E_1, E_2, E_3	various computational errors
z	longitudinal axis of the pipe
ρ	fluid density ($\mathrm{kg/m^3}$)
μ	Poisson's ratio of the pipeline material
τ	pipe wall thickness (m)
ψ	pipeline inclination angle with the horizontal (deg)
θ	a constant parameter
Δt	time step (s)
Δz	space step along pipe axis (m)
N	number of nodes of the pipeline

Subscripts

i	nodal index

Superscripts

t time index

ACKNOWLEDGEMENTS

The authors wish to acknowledge the support of King Fahd University of Petroleum & Minerals (KFUPM) as well as the support provided by King Abdulaziz City for Science and Technology (KACST) through research grant AR-17-34.

REFERENCES

1. Stoner MA. Analysis and control of unsteady flows in natural gas piping systems. *Journal of Basic Engineering, Transactions of the ASME* 1969; 331–340.
2. Yow W. Numerical error on natural gas transient calculations. *Journal of Basic Engineering, Transactions of the ASME* 1972; 422–428.
3. Thompson WC, Skogman KD. The application of real time flow modeling to pipeline leak detection. *Journal of Energy Resources Technology, Transactions of the ASME* 1983; **105**:538–541.
4. Liou JCP, Tian J. Leak detection-transient flow simulation approaches. *Journal of Energy Resources Technology, Transactions of the ASME* 1995; **117**:243–248.
5. Choy FK, Braun MJ, Wang HS. Transient pressure analysis in piping networks due to valve closing and outlet pressure pulsation. *Journal of Pressure Vessel Technology* 1996; **118**:315–325.
6. Watters GZ. *Modern Analysis and Control of Unsteady Flow in Pipelines*. Butterworths Publishers: London, 1984.
7. Cheney W, Kincaid D. *Numerical Mathematics and Computing* (4th Edition). Brooks/Cole Publishing Co: Pacific Grove, CA, 1999.
8. Hoffman JD. *Numerical Methods for Engineers and Scientists* (International Editions). McGraw-Hill, Inc: New York, 1993.
9. Thorley AD, Tiley CH. Unsteady and transient flow of compressible fluids in pipelines—a review of theoretical and some experimental studies. *The International Journal of Heat and Fluid Flow* 1987; **8**(1):3–15.
10. Osiadacz AJ. *Simulation and Analysis of Gas Networks*. E. & F. N. Spon Ltd.: London, 1987.

